

Graded contractions of the \mathbb{Z}_2^3 -gradings on the exceptional Lie algebras coming from octonions

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Abstract.

Given a G -graded Lie algebra \mathcal{L} , a graded contraction on the grading is a map $\varepsilon: G \times G \rightarrow \mathbb{F}$ such that, by modifying the Lie bracket according to $[x, y]^\varepsilon = \varepsilon(g, h)[x, y]$ if $x, y \in \mathcal{L}$ with degrees g and h respectively, the resulting product still preserves the Lie algebra structure. In this way, such maps allow us to modify the original Lie algebra and obtain a new one with different properties: solvable, nilpotent, with a more abelian structure, or even reductive. Recently, the graded contractions on the \mathbb{Z}_2^3 -grading on the complex Lie algebras \mathfrak{g}_2 , \mathfrak{b}_3 and \mathfrak{d}_4 induced by the octonions have been completely classified up to isomorphism [1, 2]. This classification was carried out using combinatorial techniques on their supports $Supp(\varepsilon) := \{(g, h) \in G \times G : \varepsilon(g, h) \neq 0\}$.

This combinatorial framework is extended to the case of generic graded contractions of the same grading group in order to obtain new examples applicable to the Tits construction. This leads to 860 \mathbb{Z}_2^3 -graded Lie algebras of dimensions 52, 78, 133, and 248 are obtained as graded contractions of the \mathbb{Z}_2^3 -gradings on the exceptional Lie algebras (excluding \mathfrak{g}_2) arising from the octonions in [3]. Moreover, the resulting algebras are distinguished by their Levi decompositions, the derived series of their radicals, their centers, and other structural features.

References

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- [3] F. Cuenca Carrégalo, C. Draper, and T. L. Meyer, *Graded contractions of the \mathbb{Z}_3^2 -gradings on the exceptional Lie algebras coming from octonions*, preprint arXiv:2508.02245, 2025.