On Spaces of Nilpotent Matrices and a Problem of Albert

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Abstract.

In the past half-century, M. Gerstenhaber established the seminal results relating nilpotent subspaces of matrices over fields [2, 3, 4, 5]. These works emerge in connection with a problem in finite-dimensional commutative algebras proposed by A. Albert [1]. Here, we will show new results about space of nilpotent $n \times n$ matrices over arbitrary fields. In [8], the author describes, up to similarity, the maximal nilpotent linear subspaces of $M_4(\mathbb{C})$. The main result of [6] consists of the classification, up to similarity, of all maximal nilpotent linear subspaces of $M_4(F)$, where F is a field with at least three elements. In particular, we find a new maximal nilpotent subspace of $M_4(\mathbb{C})$ which is not similar to the subspaces given in [8]. In [7] we describe all two-dimensinal nilpotent linear subspaces of $M_5(\mathbb{C})$ that attain maximal rank. I expect that these results will allow us to describe every maximal nilpotent linear subspace of $M_5(\mathbb{C})$.

References

[1] A.A. Albert, A theory of power-associative commutative algebras, *Trans. Amer. Math. Soc.* 69 (1950), 503–527.

[2] M. Gerstenhaber, On nilalgebras and linear varieties of nilpotent matrices I, *Amer. J. math.* 80 (1958), 614–622.

[3] M. Gerstenhaber, On nilalgebras and linear varieties of nilpotent matrices II, *Duke Math. J.* 27 (1960), 21–31.

[4] M. Gerstenhaber, On nilalgebras and linear varieties of nilpotent matrices III, *Ann. of Math.* 70 (1959), 167–205.

[5] M. Gerstenhaber, On nilalgebras and linear varieties of nilpotent matrices IV, *Ann. of Math.* 75 (1962), 382–418.

[6] J.C.G. Fernandez and Claudia I. Garcia, , *Nilpotent linear spaces in* $M_4(F)$, Linear Multilinear Algebra, 73 (2025), no. 16, 3638–3651. https://doi.org/10.1080/03081087.2025.2524019.

[7] J.C.G. Fernandez and Claudia I. Garcia, *Nilpotent linear spaces in* $M_5(\mathbb{C})$, Journal of Algebra and Computation, to appear.

[8] M. A. Fasoli, *Classification of nilpotent linear spaces in* $M(4;\mathbb{C})$*,* Comm. Algebra 25 (1997), no. 6, 1919–1932.