Generalized Lie structures of associative graded algebras with special involutions.

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Abstract.

Every associative algebra A gives rise to a Lie algebra A^- by replacing the associative product ab by the commutator $ab - ba$. The ideals of A^- are usually called Lie ideals of A. The study of the Lie ideal structure of A dates back to the 50s with the works of Baxter [1] and Herstein [4, 5] in associative rings. More precisely, they investigated the Lie ideal structure of an associative simple ring A, as well as, the Lie ideal structure of K and $[K, K]$, for K the skew symmetric elements of a simple ring with involution and characteristic not 2. Their results were generalized in several ways: Erickson [6] extended them to prime rings with involution which are 2-torsion-free, while Lanski and Montgomery [7] dealt with prime rings of characteristic 2. All these results are known nowadays as Herstein's Lie Theory.

In the past few years, Herstein's Lie Theory has been extended to other algebraic structures like superalgebras [8, 9, 10, 11] and Lie color algebras [2, 12, 14]. Here, we focus our attention on the skew symmetric elements K of an (ε, G)-Lie color algebra with an ε-involution. See [3, 13] for some background on Lie color algebras. More specifically, we investigate the ε-Lie structure of K and $[K, K]$ and we explore the relationship with the (associative) ideals of A.

References

[1] W. E. Baxter, Lie simplicity of a special case of associative rings II, Trans. Amer. Math. Soc. 87 (1958), 63–75.

[2] Y. Bathurin, D. Fischman, S. Montgomery, On the generalized Lie structure of associative algebras, Israel J. Math. 96 (1996), 27-48.

[3] Y. Bahturin, A. Mikhalev, V. Petrogradskii, M. Zaicev, *Infinite Dimensional Lie Superalgebras* Expos. Math., Vol. 7, de Gruyter, Berlin, 1992.

[4] I. N. Herstein, “Topics in Ring Theory", Univ. of Chicago Press, Chicago, 1969.

[5] I. N. Herstein, “Rings with Involution", Univ. of Chicago Press, Chicago, 1976.

[6] T. E. Erickson, The Lie structure in prime rings with involution. J. Algebra 21 (1972), 523–534.

[7] C. Lanski, M. Montgomery, Lie structure or prime rings of characteristic 2. Pacific J. Math. 48(1) (1972) 117-186

[8] C. Gómez-Ambrosi, I. P. Shestakov, On the Lie structure of the skew elements of a simple superalgebra with superinvolution, J. Algebra 208 (1998), 43-71.

[9] C. Gómez-Ambrosi, J. Laliena, I. P. Shestakov, On the Lie structure of the skew elements of a prime superalgebra with superinvolution, Comm. Algebra 28 (7) (2000), 3277-3291

[10] J. Laliena, S. Sacristán, Lie structure in semiprime superalgebras with superinvolution, J. Algebra 315 (2007), 751-760.

[11] F. Montaner, On the Lie structure of associative superalgebras, Comm. Algebra 26 (7) (1998), 2337-2349.

[12] S. Montgomery, Constructing simple Lie superalgebras from associative graded algebras, J. Algebra 195 (1997), 558-579.

[13] M. Scheunert, Generalized Lie algebras, J. Math. Phys. 20 (1979), 712–720.

[14] K. Zhao, Simple Lie color algebras from graded associative algebras, J. Algebra 269 (2003), 439–455.