Cohomological rigidity of solvable Lie algebras of maximal rank

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Abstract.

The second adjoint cohomology group $H^2(\mathfrak{g},\mathfrak{g})$ occupies a central place in study of Lie algebras, linking cohomological algebra, geometric deformation theory, and the structure of the algebraic variety of Lie laws. Despite its importance, obtaining explicit criteria for the vanishing or non-vanishing of this invariant remains highly nontrivial.

We discuss on the developing of a general framework for computing the second adjoint cohomology group of solvable Lie algebras of the form semidirect product of $\mathcal{R}_{\mathcal{T}} = \mathcal{N} \rtimes \mathcal{T}$, where $\mathcal{N}$ is a nilpotent Lie algebra of maximal rank and $\mathcal{T}$ is a maximal torus acting diagonally on $\mathcal{N}$. Building on and significantly extending the classical Leger–Luks' method [1], we derive explicit sufficient conditions for the vanishing of $H^2(\mathcal{R}_{\mathcal{T}},\mathcal{R}_{\mathcal{T}})$. Our results apply to broad families of solvable Lie algebras, including all cohomologically rigid algebra of the form $\mathcal{R}_{\mathcal{T}}$ appearing in low-dimensional (up to 9) and maximal solvable extensions of the well-known model filiform and model nilpotent Lie algebras.

In addition, we establish sufficient conditions for the non-vanishing of the second adjoint cohomology, thereby clarifying the precise situations in which rigidity fails. The interaction between the root configuration of the nilradical and the pattern of $\mathcal{T}$-invariant cocycles plays a key role in these results. Our results also yield a conjectural lower bound on $\dim H^2(\mathcal{R}_{\mathcal{T}},\mathcal{R}_{\mathcal{T}})$.

References

[1] G. Leger and E. Luks, *Cohomology theorems for Borel-like solvable Lie algebras in arbitrary characteristic*, Canad. J. Math. (1972), no. 24, 1019-1026.