

# Associative representations of evolution algebras of dimension two.

Iván Ruiz Campos

**Yolanda Cabrera Casado** (yolandacc@uma.es)  
Universidad de Málaga

**Maria Inez Cardoso Gonçalves** (maria.inez@ufsc.br)  
Universidade Federal de Santa Catarina.

**Daniel Gonçalves** (daemig@gmail.com)  
Universidade Federal de Santa Catarina.

**Dolores Martín Barquero** (dmartin@uma.es)  
Universidad de Málaga

**Cándido Martín González** (candido\_m@uma.es)  
Universidad de Málaga

**Iván Ruiz Campos** (ivaruicam@uma.es)  
Universidad de Málaga

## **Abstract.**

In this talk, we establish a connection between evolution algebras of dimension two and Hopf algebras, via the algebraic group of automorphisms of an evolution algebra. Initially, we describe the Hopf algebra associated with the automorphism group of a 2-dimensional evolution algebra. Subsequently, we center our study in the notion of associative representation for nonassociative algebras, as introduced by Shestakov and Kornev in [2]. Furthermore, for a 2-dimensional evolution algebra  $A$  over a field  $K$ , we detail the relation between the algebra associated with the (tight) universal associative and commutative representation of  $A$ , referred to as the (tight)  $p$ -algebra, and the corresponding Hopf algebra,  $\mathcal{H}$ , representing the affine group scheme  $\text{Aut}(A)$ .

Our analysis involves the computation of the (tight)  $p$ -algebra associated with any 2-dimensional evolution algebra, whenever it exists. We find that  $\text{Aut}(A) = 1$  if and only if there is no faithful associative and commutative representation for  $A$ . Moreover, there is a faithful associative and commutative representation for  $A$  if and only if  $\mathcal{H} \not\cong K$  and  $\text{char}(K) \neq 2$ , or  $\mathcal{H} \not\cong K(\epsilon)$  (the dual numbers algebra) and  $\mathcal{H} \not\cong K$  in case of  $\text{char}(K) = 2$ . Furthermore, if  $A$  is perfect and has a faithful tight  $p$ -algebra, then this  $p$ -algebra is isomorphic to  $\mathcal{H}$  (as algebras). Finally, we derive implications for arbitrary finite-dimensional evolution algebras.

## References

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