Nonassociative Methods in Pure Matrix Algebras

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Abstract.

A central problem in invariant theory is the description of generators and relations for the algebra of invariants of d-tuples of $n \times n$ matrices under simultaneous conjugation. While in general the generators and relations have been known since the work of Procesi and Razmyslov, finding a minimal set of generators and theird defining relations remains a formidable computational challenge as n increases.

In this talk, we present recent results joint with X. García-Martínez and F. Eshmatov, that utilize nonassociative structures, specifically the Poisson algebra structure, to navigate this complexity. By treating the algebra of invariants as a Poisson algebra, we demonstrate how higher-degree relations can be systematically generated from lower-degree ones. This methodology has allowed us to solve the problem for 4×4 matrices, where we identify a minimal set of just 8 relations that generate the associative ideal of 105 polynomials. We further discuss the applications of these results to the geometry of Calogero-Moser spaces and the invariant commuting variety of matrices.