Computing the regularity of the deficiency modules

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Abstract.

The regularity is one of the most studied invariants of a finitely generated graded module M over a polynomial ring R in the last years in Commutative Algebra. At first instance, it provides a bound for the degrees of the generators of M and it is also related with the complexity of its minimal graded free resolution. In the case of the graded deficiency modules $K^j(R/I)$ of a graded quotient ring R/I there is also a relation with the behaviour of the h-vector of R/I. Explicit computations of the regularity of the deficiency modules are scarce and, in general, only large bounds are known except for a celebrated result by Kumini and Murai in [3] for the monomial case, which states that $\operatorname{reg}(K^j(R/I)) \leq j$. In this talk I will explain how to obtain upper bounds for the regularity of graded deficiency modules building upon the spectral sequence formalism developed by Àlvarez Montaner, Boix and Zarzuela in [1]. This spectral sequence formalism allows us not only to recover Kummini–Murai's upper bound for monomial ideals, but also to extend it for other types of rings, which include toric face rings and some binomial edge rings, producing new upper bounds for the regularity of graded deficiency modules of this type of rings.

Based on a joint work with A. F. Boix [2].

References

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[3] M. Kummini, and S. Murai, *Regularity of canonical and deficiency modules for monomial ideals*, Paciific J. Math. 377 (2011), no. 2, 377–383.