Spectral Galois Theory for the KdV Variational Equation

M. A. Zurro

Maria-Angeles Zurro (mangeles.zurro@uam.es)
Universidad Autónoma de Madrid, Madrid, Spain

Abstract.

This talk addresses the Galoisian study of integrability for nonlinear partial differential equations, a problem that remains largely open within the framework of differential Galois theory. Our approach is motivated by the classical example of the Korteweg–de Vries (KdV) equation [4], whose integrability is understood through its associated Lax pair—a pair of differential operators for which the KdV equation arises as the compatibility condition.

A foundational step in this direction was taken by Morales-Ruiz, Rueda and Zurro [1], who developed a spectral Galois theory for the Schrödinger operator with stationary KdV potential. More recently, an alternative categorical framework has been proposed by Tomašić [3]. Building on these contributions, and on the algebro-geometric third-order operators studied by Rueda and Zurro [2], we present a Galoisian approach to the variational equation associated with a KdV-type evolution equation in dimension 1+1, exploiting the spectral techniques arising from third-order operator theory.

As a central application, we carry out explicit computations of solutions to the variational equation of KdV around a cnoidal wave. These calculations have been performed and verified using the computer algebra system Maple.

This work is part of an ongoing collaboration with J. J. Morales-Ruiz and J. P. Ramis.

References

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