Explicit canonical cycle at the virtual cohomological dimension of $\mathrm{SL}_n(\mathbb{Z})$ through Voronoi complex

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Abstract.

We construct an explicit canonical cycle in the top-dimensional homology of the Voronoi complex associated with an arithmetic group. This cycle relates to the cohomology of $\mathrm{SL}_n(\mathbb{Z})$ with rational coefficients at the virtual cohomological dimension. It had previously been identified in computational works and conjectured to provide an intrinsic generator.

Our approach relies on a geometric rigidity property of the Voronoi tessellation. More precisely, we show that codimension-one facets split into two types: non-self-intersecting facets, whose contributions cancel pairwise between neighbouring top cells, and self-intersecting facets, whose cancellation is internal to the stabilizer of a single top cell. Together with the connectedness of the Voronoi graph, this yields a canonical non-trivial cycle of the form

$$\sum_{\sigma \in \Sigma_{d(n)}} \frac{1}{|\Gamma_\sigma|}\, \sigma.$$

Furthermore, we formulate an abstract framework for polyhedral tessellations of convex cones under group actions, clarifying the mechanism behind the construction of such cycles. As a consequence, we prove that this canonical cycle generates the top homology group of the Voronoi complex with rational coefficients.

References

[1] A. de la Torre Durán, *Explicit canonical cycle at the virtual cohomological dimension of* $\mathrm{SL}_n(\mathbb{Z})$ *through Voronoi complex*, preprint arXiv:2604.03743, 2026.