ManoloFest
Categorical, homological and computational methods in Algebra

Automorphism group schemes on Kantor Pairs related to associative algebras

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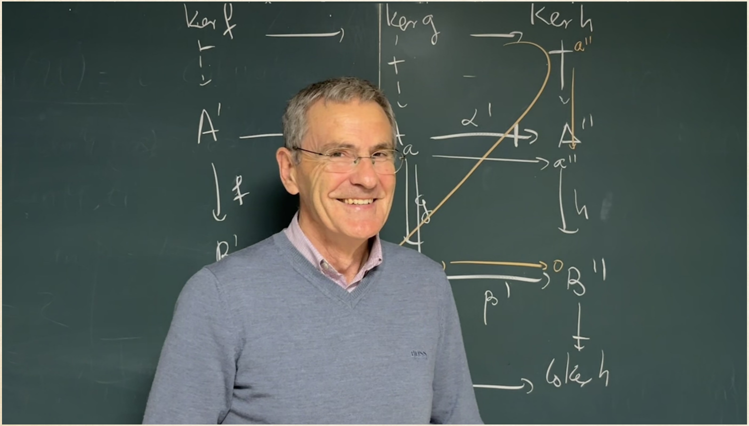

- $(\mathcal{A}, -)$ an algebra with involution and unit 1 over a field with $\mathrm{char}(\mathbb{F}) \neq 2$
- $V_{x,y}z = (x\bar{y})z + (z\bar{y})x - (z\bar{x})y$
- $T_z = V_{1,z}$

Definition: $(\mathcal{A}, -)$ is a **structurable algebra** if:

- $[T_z, V_{x,y}] = V_{T_z x,y} - V_{x,T_{\bar{z}}y}$
- $(x - \bar{x}, y, z) = (y, x - \bar{x}, z)$

- $\mathcal{S} = \{a \in \mathcal{A} \mid \bar{a} = -a\}$

- $\mathfrak{Instr}(\mathcal{A}, -) = \langle V_{x,y} \mid x, y \in \mathcal{A}\rangle$

$$\mathcal{K}(\mathcal{A}, -) = \underbrace{\mathcal{S}}_{-\bar{2}} \oplus \underbrace{\mathcal{A}}_{-\bar{1}} \oplus \underbrace{\mathfrak{Instr}(\mathcal{A}, -)}_{-\bar{0}} \oplus \underbrace{\mathcal{A}}_{-\bar{1}} \oplus \underbrace{\mathcal{S}}_{\bar{2}}$$

Definition: A **Kantor pair** consists on:

- A pair of vector spaces $(\mathcal{V}^+, \mathcal{V}^-)$
- A pair of trilinear products

$$\{\cdots\}^\sigma : \mathcal{V}^\sigma \times \mathcal{V}^{-\sigma} \times \mathcal{V}^\sigma \to \mathcal{V}^\sigma$$

satisfying some identities ($\sigma = \pm$).

Definition: An **automorphism** of the Kantor pair $(\mathcal{V}^+, \mathcal{V}^-)$ is a pair (φ^+, φ^-):

$$\varphi^\sigma : \mathcal{V}^\sigma \to \mathcal{V}^\sigma$$

satisfying:

$$\varphi^\sigma\{x, y, z\}^\sigma = \{\varphi^\sigma(x), \varphi^{-\sigma}(y), \varphi^\sigma(z)\}^\sigma$$

The main example is:

- $(\mathcal{A}, -)$ is a structurable algebra,
- $\mathcal{V}^+ = \mathcal{V}^- = \mathcal{A}$
- $\{x, y, z\}^\sigma = V_{x,y}z \ (\sigma = \pm)$

We denote it $\mathcal{V}_{(\mathcal{A}, -)}$

$\mathrm{char}(\mathbb{F}) \neq 3$

- $\mathbf{Aut}(\mathcal{V}_{(\mathcal{A},\,-\,)}) \cong \mathbf{Aut}(\mathcal{K}(\mathcal{A},\,-\,))$
- $\mathbf{Stab}_{\mathbf{Aut}(\mathcal{V}_{(\mathcal{A},\,-\,)})}(1^+, 1^-) \cong \mathbf{Aut}(\mathcal{A},\,-\,)$
- If $(\mathcal{A},\,-\,)$ central simple:

$$\mathrm{Der}(\mathcal{V}_{(\mathcal{A},\,-\,)}) \cong \mathrm{Lie}(\mathbf{Aut}(\mathcal{V}_{(\mathcal{A},\,-\,)})) \cong \mathfrak{Instr}(\mathcal{A},\,-\,)$$

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Theorem: (Allison 78, Smirnov 90) Every central simple structurable algebra over a field of characteristic $\neq 2, 3, 5$ is isomorphic to either:

- a Jordan algebra with the identity involution,
- an associative algebra with involution,
- a structurable algebra related to a hermitian form,
- a twisted form of a tensor product of Hurwitz algebras
- a twisted form of an algebra constructed form an admisible triple
- the Smirnov algebra $T(\mathcal{C})$

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$\mathbb{F} = \overline{\mathbb{F}}$

Theorem: An associative algebra with involution is isomorphic to either:

- $\mathcal{M}_n(\mathbb{F})$ with the transposition
- $\mathcal{M}_{2m}(\mathbb{F})$ with the symplectic involution:

$$\widehat{X} = \Omega X^t \Omega^{-1}$$

  with

$$\Omega = \begin{pmatrix} 0 & \mathrm{I}_m \\ -\mathrm{I}_m & 0 \end{pmatrix}$$

- $\mathcal{M}_n(\mathbb{F}) \times \mathcal{M}_n(\mathbb{F})^{op}$ with the exchange involution:

$$\mathrm{ex}(X, Y) = (Y, X)$$

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Theorem: An associative algebra with involution is isomorphic to either:

- **1st Kind** $\mathcal{M}_n(\mathbb{F})$ with the transposition
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- **2nd Kind** $\mathcal{M}_n(\mathbb{F}) \times \mathcal{M}_n(\mathbb{F})^{op}$ with the exchange involution:

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- $\mathcal{V}_R := \mathcal{V} \otimes_{\mathbb{F}} R$

Some affine group schemes

- $\mathcal{V}_R := \mathcal{V} \otimes_{\mathbb{F}} R$
- $\mathbf{GL}_n(R) = \{\varphi : \mathbb{F}_R^n \to \mathbb{F}_R^n | \varphi \text{ invertible } \}$

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- For $a \in \mathcal{A}^\times$, $\widetilde{L}_a = (L_a, L_{\bar{a}^{-1}}) \in \mathrm{Aut}(\mathcal{V}_{(\mathcal{A},\,-\,)})$
- For $a \in \mathrm{Sim}(\mathcal{A},\,-\,)$, $a\bar{a} = \mu(a)1$

$$\Rightarrow \widehat{R}_a = (R_a, \tfrac{1}{\mu(a)} R_a) \in \mathrm{Aut}(\mathcal{A},\,-\,)$$

Theorem: If $(\mathcal{A}, -) \cong (\mathcal{M}_n(\mathbb{F}), {}^t)$ or $(\mathcal{M}_{2m}(\mathbb{F}), \hat{\ })$:

$$\mathbf{Aut}(\mathcal{V}_{(\mathcal{A},\, -)}) \cong \frac{\mathbf{G}_m(\mathcal{A}) \times \mathbf{Sim}(\mathcal{A}, -)}{\mathbf{G}_m}$$

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Involution of the 1st Kind

Little outline of the proof

- Conjugate invertible $\Rightarrow$ invertible.
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- $\varphi = \widetilde{L}_{ca} \circ \widehat{R}_{c^{-1}}$, $ca \in \mathbf{G}_m(\mathcal{A})$, $c \in \mathbf{Sim}(\mathcal{A}, -)$.

Corollary:

- $\mathbf{Aut}(\mathcal{V}_{(\mathcal{M}_n(\mathbb{F}),\, ^t)}) \cong \frac{\mathbf{GL}_n \times \mathbf{GO}_n}{\mathbf{G}_m}$

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Involution of the 2nd Kind

Theorem: If $(\mathcal{A}, -) \cong (\mathcal{M}_n(\mathbb{F}) \times \mathcal{M}_n(\mathbb{F})^{op}, \mathrm{ex})$ and $n \neq 1$ or $\mathrm{char}(\mathbb{F}) \neq 3$:

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Thank you!

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satisfying some identities:(T,T) with $\{\cdots\}^{+} = \{\cdots\}^{-} = \{\cdots\}$ is a Kantor pair.

Bonus track

Theorem: If $(\mathcal{A}, -) \cong (\mathcal{M}_n(\mathbb{F}), {}^t)$ or $(\mathcal{M}_{2m}(\mathbb{F}), \hat{\ })$:

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Corollary: If $(\mathcal{A}, -) \cong (\mathcal{M}_n(\mathbb{F}), {}^t)$:

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