

EXPERIMENTAL MATHEMATICS USING SYMBOLIC COMPUTATION TO ANALYZE CONJECTURES INVOLVING MATRICES

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... PROBLEMS ...
... SOME ANSWERS ...
... AND ...
... EVEN MORE
PROBLEMS ...

ALWAYS WITH AN EXPERIMENTAL
& COMPUTATIONAL FLAVOUR

- ✱ Bohemian Correlation Matrices.
- ✱ Computing the spread.
- ✱ Eigenvalues of QM -matrices.
- ✱ No conclusions but

ON BOHEMIAN CORRELATION MATRICES

WITH THE HELP OF
R. CORLESS, N. HIGHAM, E. Y. S CHAN,
J. SENDRA & R. SENDRA

**ON THE NUMBER OF
CORRELATION MATRICES
IN THE SET OF $N \times N$
BOHEMIAN MATRICES
WHEN N IS FIXED**

$$\#(\text{BCM}_{n:\{0,1\}})$$

Data:

- Population: 0 and 1 or -1 , 0 and 1 or -1 and 1 .
- Type of matrices: $n \times n$ Bohemian Matrices.
- $\mathbf{BM}_{n:\{0,1\}}$ or $\mathbf{BM}_{n:\{-1,0,1\}}$ or $\mathbf{BM}_{n:\{-1,1\}}$.

Output:

- For every n , compute the number of correlation matrices in the set $\mathbf{BM}_{n:\{0,1\}}$: $\mathbf{BCM}_{n:\{0,1\}}$

An $n \times n$ symmetric matrix A is a **correlation matrix** if

- It has ones on the diagonal.
- All its eigenvalues are nonnegative.

Applications:

- Population in the closed interval $[-1,1]$.

Problem at hand

Looking for $\{0, 1\}$ correlation matrices.

n	total matrices	# correl	proportion correl
3	8	5	6.25e-1
4	64	15	2.34e-1
5	1024	52	5.08e-2
6	32768	203	6.20e-3
7	2097152	877	4.18e-4

Nick Higham's table from Manchester Bohemian Workshop, 2018



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Out of the $2^{\text{binomial}(8,2)} = 268.435.456$ possibilities, there are only **4140** correlation matrices giving a proportion of $1.54\text{e-}5$. Computing time was 24 hours and a few minutes.

$$\#(\text{BCM}_{8:\{0,1\}}) = 4140$$



THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES[®]

founded in 1964 by N. J. A. Sloane

5, 15, 52, 203, 877, 4140

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(Greetings from [The On-Line Encyclopedia of Integer Sequences!](#))

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Displaying 1-10 of 40 results found.

page 1 [2](#) [3](#) [4](#)

Sort: relevance | [references](#) | [number](#) | [modified](#) | [created](#) Format: long | [short](#) | [data](#)

[A000110](#) Bell or exponential numbers: number of ways to partition a set of n labeled elements.
(Formerly M1484 N0585)

+30
954

1, 1, 2, **5, 15, 52, 203, 877, 4140**, 21147, 115975, 678570, 4213597, 27644437,
190899322, 1382958545, 10480142147, 82864869804, 682076806159, 5832742205057,
51724158235372, 474869816156751, 4506715738447323, 44152005855084346, 445958869294805289,
4638590332229999353, 49631246523618756274 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

$$B_n = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^n}{k!} \quad B_{n+1} = \sum_{k=0}^n \binom{n}{k} B_k \quad \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n = e^{e^x - 1}$$

The solution

$a(n+1)$ is the number of (symmetric) positive semidefinite $n \times n$ 0-1 matrices. These correspond to equivalence relations on $\{1, \dots, n+1\}$, where matrix element $M[i, j] = 1$ if and only if i and j are equivalent to each other but not to $n+1$. - [Robert Israel](#), Mar 16 2011

How computations were performed?

- First approach with Matlab for computing eigenvalues giving wrong results for $n = 7$ due to precision problems.
- Second approach: Maple but avoiding eigenvalue computation.

CORRELATION MATRICES: CHARACTERISATION

The characterisation

Theorem

Let A be a symmetric $n \times n$ matrix. Then we have:

- *A is positive definite $\Leftrightarrow D_k > 0$ for all leading principal minors*
- *A is negative definite $\Leftrightarrow (-1)^k D_k > 0$ for all leading principal minors*
- *A is positive semidefinite $\Leftrightarrow \Delta_k \geq 0$ for all principal minors*
- *A is negative semidefinite $\Leftrightarrow (-1)^k \Delta_k \geq 0$ for all principal minors*

Too many principal minors to check: $\binom{n}{k}$ for $1 \leq k \leq n$

Descartes Law of Signs

For a sequence of real numbers $b_0, b_1, \dots, b_n$, $\mathbf{Var}(b_0, b_1, \dots, b_n)$ will denote the number of sign changes in $b_0, b_1, \dots, b_n$ after dropping the zeros in the sequence.

Descartes' law of signs:

Let P be the polynomial in $\mathbb{R}[x]$:

□

$$P(x) = \sum_{k=0}^n a_k x^k.$$

The number of positive real roots of $P(x) = 0$, counted with multiplicity, is equal to

$$\mathbf{Var}(a_n, a_{n-1}, \dots, a_0) - 2k$$

for some non-negative integer k .

Descartes law of signs is exact (taking into account the multiplicities) when all roots are known to be real.

The characterisation

All eigenvalues of A are nonnegative

if and only if

A is positive semidefinite

if and only if

Alternating signs for the coefficients of the characteristic polynomial of A

Proof: A is symmetric, real and Descartes Rule of Signs



$$\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$$

$$\lambda^3 - 3\lambda^2 - (a^2 + b^2 + c^2 - 3)\lambda - 2abc + a^2 + b^2 + c^2 - 1$$

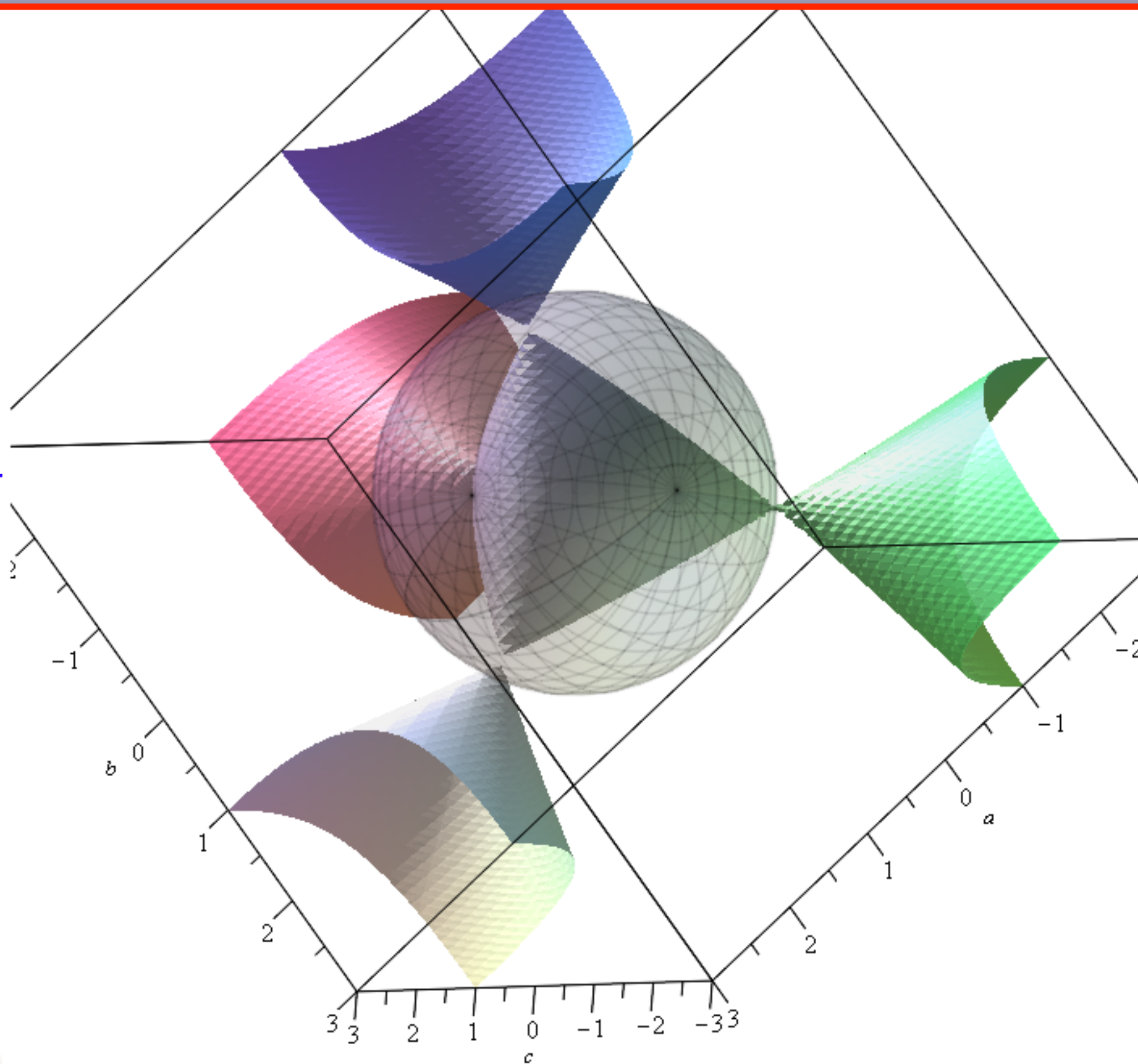
Example: n=3



$$\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$$

$$\lambda^3 - 3\lambda^2 -$$

$$-c^2 - 1$$





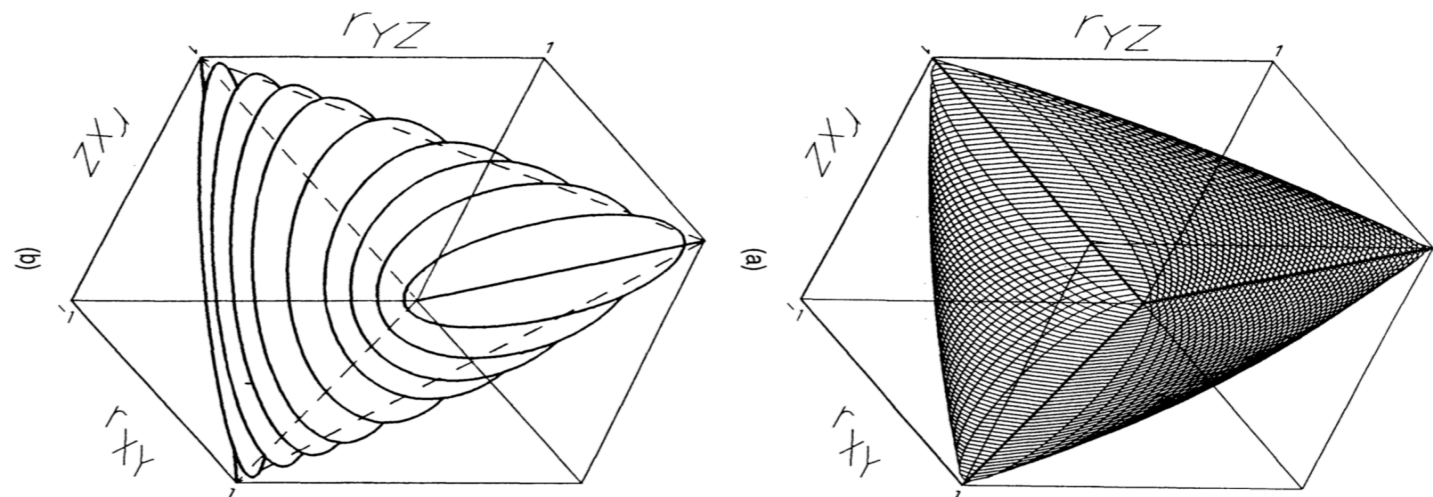
$$\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$$

$$\lambda^3 - 3\lambda^2 - (a^2 + b^2 + c^2 - 3)\lambda - 2abc + a^2 + b^2 + c^2 - 1$$

Correlation matrices. Characterisation when $n = 3$:

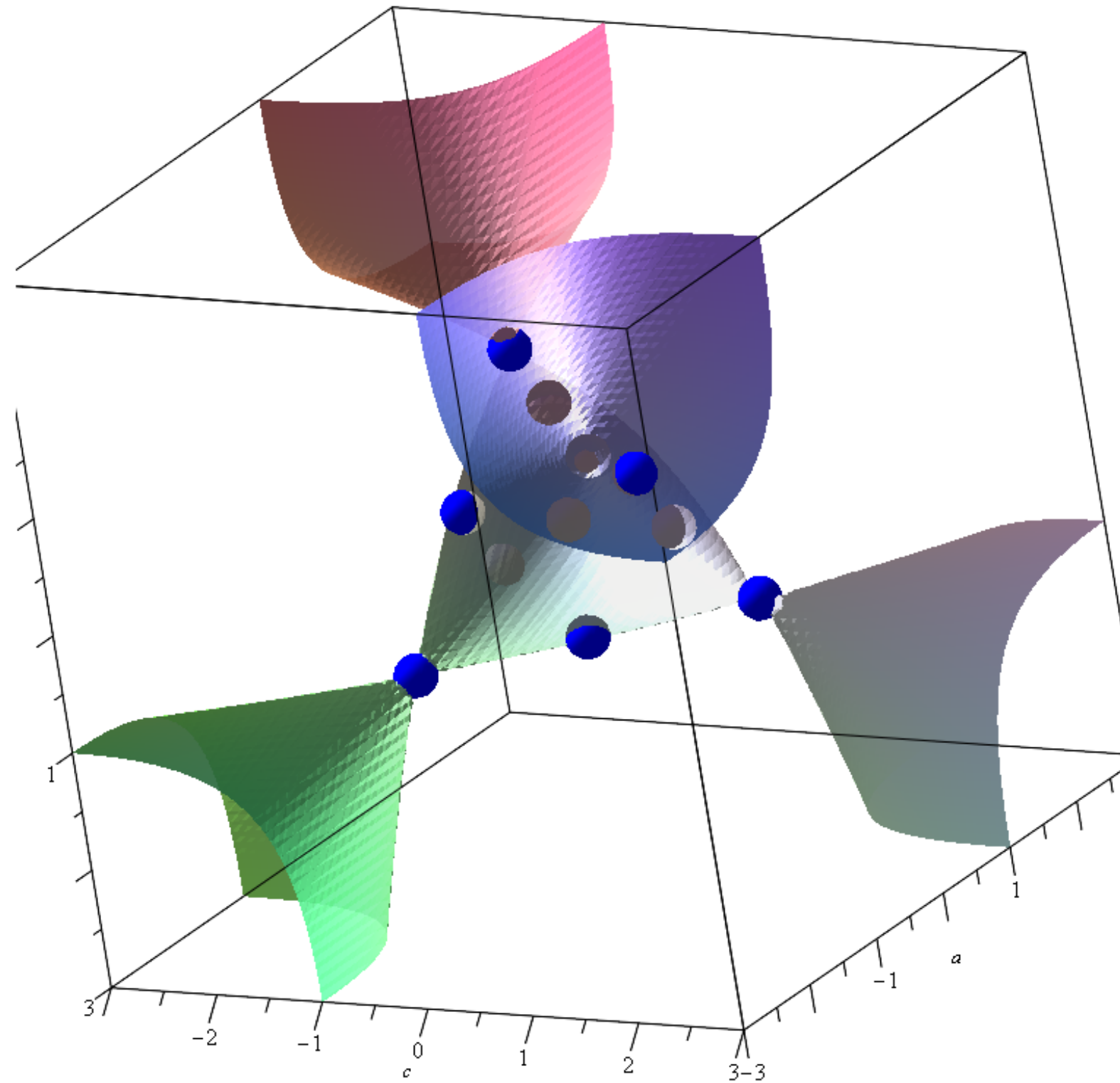
$$\left[a^2 + b^2 + c^2 - 3 \leq 0, -2abc + a^2 + b^2 + c^2 - 1 \leq 0 \right]$$

P. J. Rousseeuw and G. Molenberghs: *The Shape of Correlation Matrices*. The American Statistician 48, 276-279, 1994.



Example: n=3

$$\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$$



CBM_{3:{-1,0,1}}

$[[-1, -1, 1], [-1, 0, 0], [-1, 1, -1], [0, -1, 0], [0, 0, -1],$
 $[0, 0, 0], [0, 0, 1], [0, 1, 0], [1, -1, -1], [1, 0, 0], [1, 1, 1]]$

$$\begin{bmatrix} 1 & a & b & d \\ a & 1 & c & e \\ b & c & 1 & f \\ d & e & f & 1 \end{bmatrix}$$

Correlation matrices. Characterisation when $n = 4$:

$$-a^2 - b^2 - c^2 - d^2 - e^2 - f^2 + 6 \geq 0$$

$$-2abc - 2ade - 2bdf - 2cef + 2a^2 + 2b^2 + 2c^2 + 2d^2 + 2e^2 + 2f^2 - 4 \leq 0$$

$$a^2 f^2 - 2abef - 2acdf + b^2 e^2 - 2bcde + c^2 d^2 + 2abc + 2ade + 2bdf + 2cef - a^2 - b^2 - c^2 - d^2 - e^2 - f^2 + 1 \geq 0$$

$\#(\text{BCM}_{n:\{-1,0,1\}})$

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.....

More experiments



n	$BM_{n:\{-1,0,1\}}$	$BCM_{n:\{-1,0,1\}}$	%
3	27	11	40.74%
4	729	49	6.72%
5	59049	257	0.43%
6	14348907	1282	0.0089 %

More experiments



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11,49,257,1282

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Search: **seq:11,49,257,1282**

Sorry, but the terms do not match anything in the table.

More properties

- All matrices in $\mathbf{BCM}_{n:\{-1,0,1\}}$ except the identity are singular.
- The eigenvalues of the matrices in $\mathbf{BCM}_{n:\{-1,0,1\}}$ belong to the set

$$\{0, 1, 2, \dots, n\}$$

and there is always, at least, one multiple eigenvalue.

- All matrices in $\mathbf{BCM}_{n:\{-1,1\}}$ have the same characteristic polynomial:

$$\lambda^n - n\lambda^{n-1} = \lambda^{n-1}(\lambda - n)$$

- $\#(\mathbf{BCM}_{n:\{-1,1\}}) = 2^{n-1}$.
- $\mathbf{BCM}_{n:\{0,1\}}$ encodes Bell numbers (partitions of a set) and their characteristic polynomials encode the partitions of n .

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COMPUTING

$$\max \left\{ \text{spread}(A) : A \in S_n([a, b]) \right\}$$

N. Calkin, R. Corless, L. Gonzalez-Vega, J. Sendra & R. Sendra:
On the maximal spread of symmetric Bohemian matrices. To
appear in the IMA Journal of Numerical Analysis (2026).

Computing the maximum spread

$\mathbf{S}_n([a, b])$: $n \times n$ symmetric matrices with entrees in $[a, b]$.

$A \in \mathbf{S}_n([a, b])$ and $\lambda_1 \geq \dots \geq \lambda_n$ its eigenvalues:

$$\text{spread}(A) = \lambda_1 - \lambda_n$$

The problem: computing

$$\max \left\{ \text{spread}(A) : A \in \mathbf{S}_n([a, b]) \right\}$$

It is enough to study the case $\mathbf{S}_n([a, 1])$ with $-1 \leq a < 1$.

Computing the maximum spread

Fallat and King (2012):

$$\max \left\{ \text{spread}(A) : A \in \mathbf{S}_n([a, b]) \right\} = \max \left\{ \text{spread}(A) : A \in \mathbf{S}_n(\{a, b\}) \right\}$$

Bikorski (2022): $\max \left\{ \text{spread}(A) : A \in \mathbf{S}_n([a, b]) \right\}$

can only be attained at $\mathbf{S}_n(\{a, b\})$.

Conjecture by Fallat and King (2012):

$$\begin{aligned} & \max \left\{ \text{spread}(A) : A \in \mathbf{S}_n([a, b]) \right\} = \\ & = \max \left\{ \text{spread}(A) : A \in \mathbf{S}_n(\{a, b\}), \text{rank}(A) = 2 \right\} \end{aligned}$$

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X. Zhan (2006): the conjecture is true for $[-a, a]$ ($a > 0$).

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N. Calkin, R. Corless, LGV, J. Sendra and R. Sendra (2026),
prove that the conjecture is true for

• $\mathbf{S}_n([0, 1])$ when $n = 4, 5, 6, 7, 8$.

• $\mathbf{S}_n([a, 1])$ when $n = 2, 3, 4, 5, 6, 7$.



Computing the maximum spread

The spread of A is the biggest real root of

$$R_A(T) = \text{resultant}_\lambda(P_A(T + \lambda), P_A(\lambda))$$

where $P_A(\lambda) = \det(\lambda \mathbb{I} - A)$.

$$\#(S_n(\{a, b\})) = 2^{n(n+1)/2}$$

Computing the maximum spread

$$\#(S_6(\{0,1\})) = 2097152$$

$\text{pol} := \text{CharacteristicPolynomial}(M, T):$
 $\text{qol} := \text{resultant}(\text{subs}(T=T+\mu, \text{pol}), \text{pol}, T):$

$$\left\{ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, \right.$$

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Fallat and King (2012)

$$k = \text{round}\left(\frac{n}{a+3}\right) = 2$$

$$\text{spread}(A) = \sqrt{(a^2 + 2a - 3)k^2 + 2n(1-a)k + n^2}$$

$$= 4\sqrt{3} \sim 6.928203232.$$

Computing the maximum spread

$$\#(S_5(\{a, 1\})) = 32768 \quad \begin{array}{l} \text{pol} := \text{CharacteristicPolynomial}(M, T) : \\ \text{qol} := \text{resultant}(\text{subs}(T=T+\mu, \text{pol}), \text{pol}, T) : \end{array}$$

$$A \in S_n(\{a, 1\})$$

Given $-1 \leq a < 1$: $\text{spread}(A)$ is the biggest real root of

$$R_A(a; T) = 0$$

The conjectured maximum spread is given by

$$\sqrt{(a^2 + 2a - 3) \text{round} \left(\frac{n}{a+3} \right)^2 + 2n(1-a) \text{round} \left(\frac{n}{a+3} \right) + n^2}$$

Computing the maximum spread

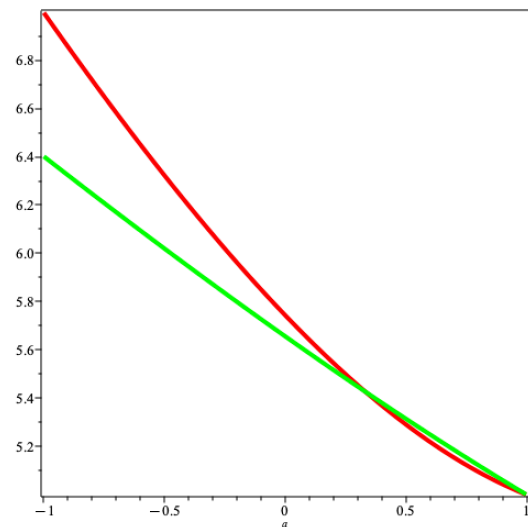
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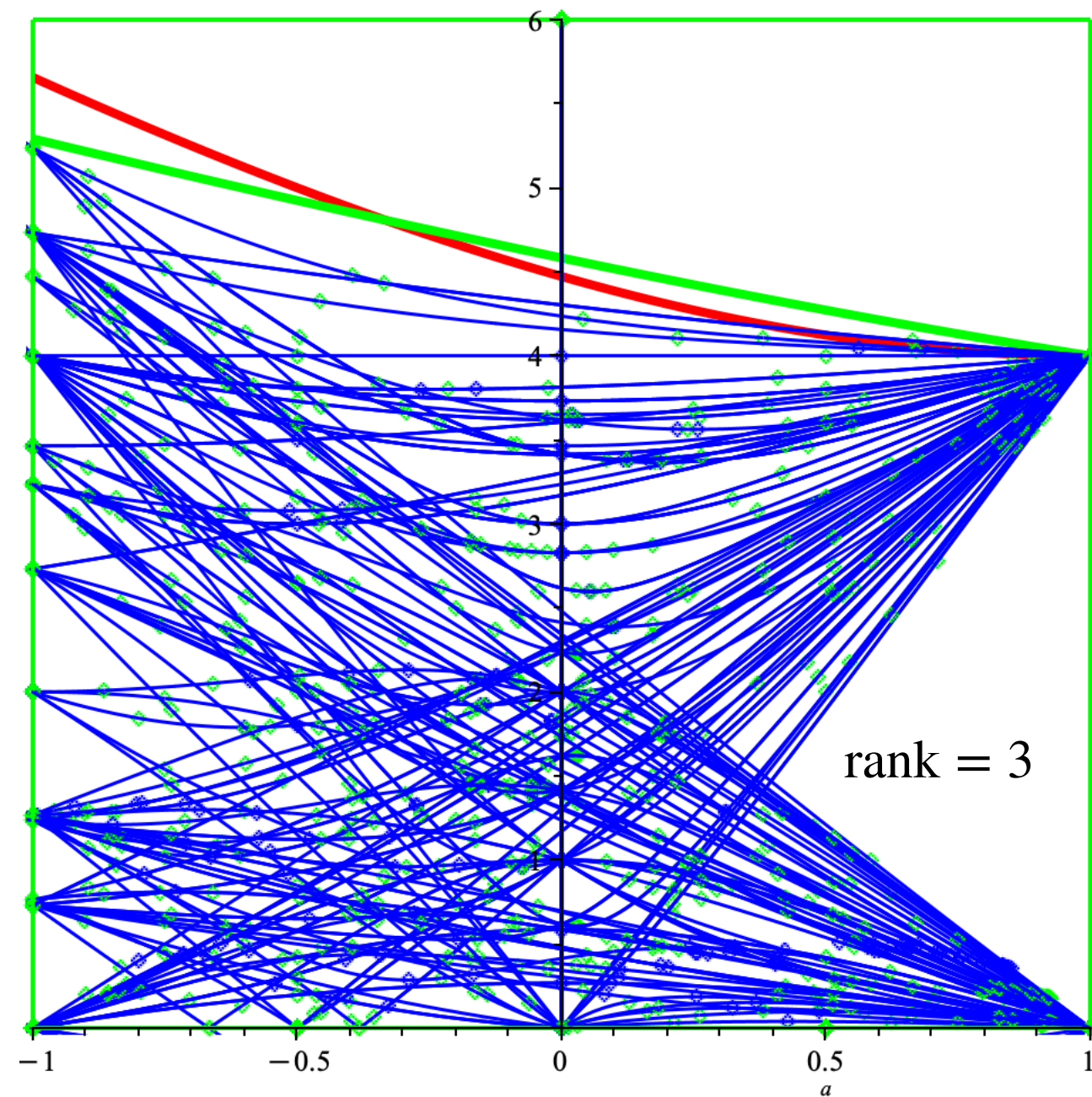
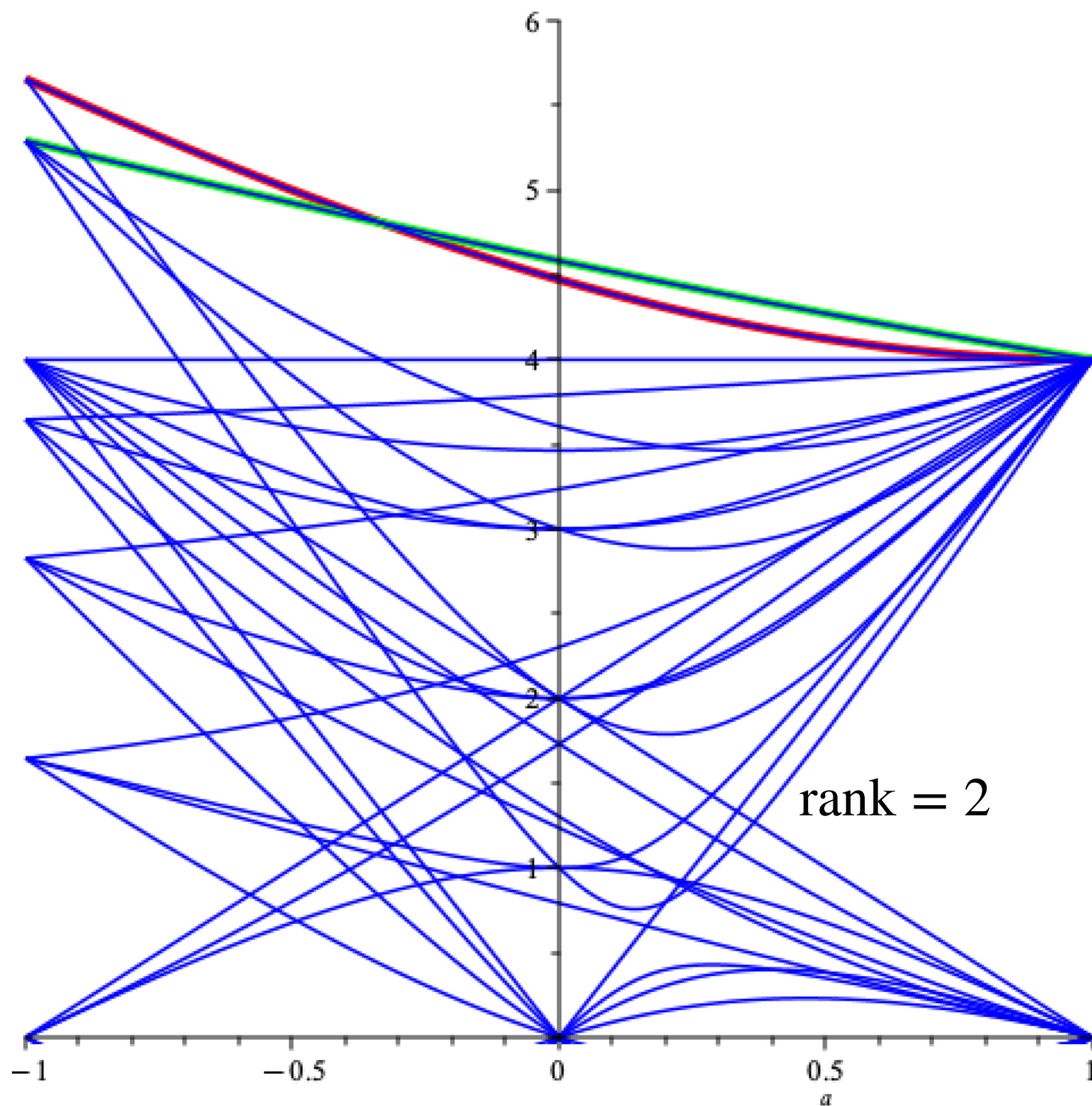


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Computing the maximum spread

Using graph isomorphism to reduce redundant computation

TABLE 1 *Reduction by canonicalization. The first column is the dimension, m . The second column is the number of different $m \times m$ symmetric matrices with each entry being either 1 or a . The third column is the number U_m of canonical graphs. The fourth column is the number of semi-canonical matrices with potentially nonzero diagonals allowing self loops. The last column contains entries from the OEIS sequence A000666, namely the numbers L_m of nonisomorphic graphs with loops.*

m	$2^{m(m+1)/2}$	U_m	$(2^m - 1)U_m$	L_m
2	8	2	6	6
3	64	4	28	20
4	1,024	11	165	90
5	32,768	34	1,054	544
6	2,097,152	156	9,828	5,096
7	268,435,456	1,044	132,588	79,624
8	68,719,476,736	12,346	3,148,230	2,208,612
9	35,184,372,088,832	274,668	140,355,348	113,743,760



We estimate that $m = 8$ would take over forty days

The conjecture is true for $S_{3q}([0,1])$

J. Breen et al: Maximum spread of graphs and bipartite graphs. Communications of the AMS, 2022.

$$A \in S_n(\{0,1\}): \text{spread}(A) \leq \frac{2n}{\sqrt{3}} \max_{i,j} \{a_{i,j}\}$$

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$$B = \begin{pmatrix} 0_{q,q} & 1_{q,2q} \\ 1_{2q,q} & 1_{2q,2q} \end{pmatrix}: \quad P_B(\lambda) = \lambda^{3q-2} (\lambda^2 - 2q\lambda - 2q^2)$$

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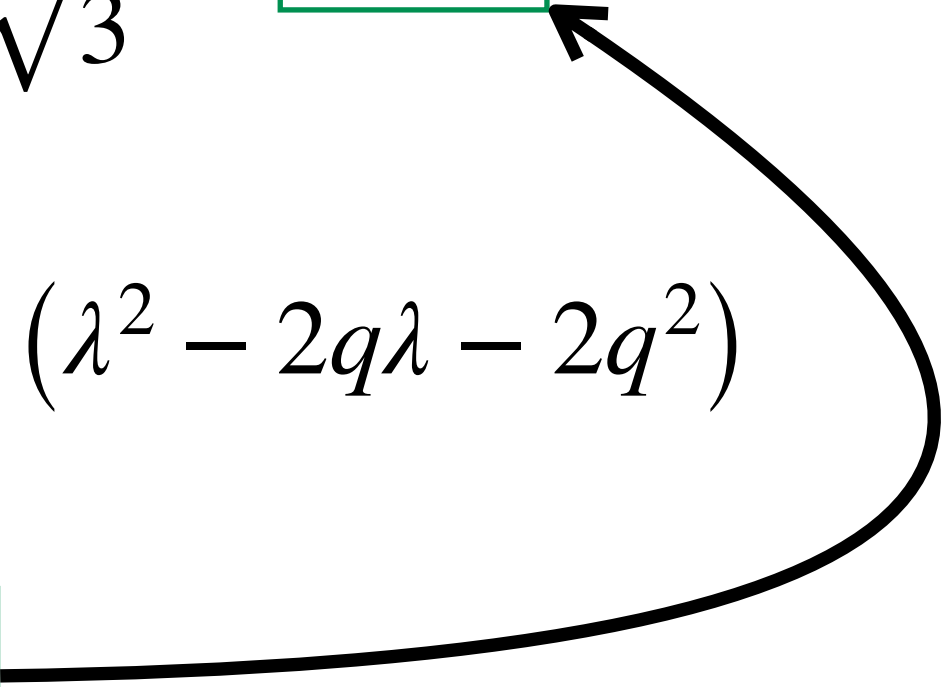
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$$B = \begin{pmatrix} 0_{q,q} & 1_{q,2q} \\ 1_{2q,q} & 1_{2q,2q} \end{pmatrix}: P_B(\lambda) = \lambda^{3q-2} (\lambda^2 - 2q\lambda - 2q^2)$$

$$\text{spread}(B) = 2\sqrt{3}q$$


ON THE EIGENVALUES OF *QM*-MATRICES

L. Gonzalez-Vega, J. Sendra & R. Sendra: Eigenvalues of Real Matrices with Prescribed Principal Minors Sign and Descartes Law of Signs. [arXiv:2305.08861](https://arxiv.org/abs/2305.08861)

A is a square matrix over $\mathbb{R}$ of size n

- ✱ A is a P -matrix if every principal minor of A is positive.
- ✱ A is a Q -matrix if the sum of the k -by- k principal minors of A is positive, for all $k = 1, \dots, n$.
- ✱ A is a PM -matrix if A^k is a P -matrix for all $k \geq 1$.
- ✱ A is a QM -matrix if A^k is a Q -matrix for all $k \geq 1$.

If A is a P -matrix then A is a Q -matrix.

If A is a PM -matrix then A is a QM -matrix.

All these matrices appear very often in many different applications:
Physics, Economy, etc.

MAIN QUESTION TO ANSWER

A is a PM -matrix if A^k is a P -matrix for all positive integers k

Are the eigenvalues of a PM -matrix necessarily positive?

D. Hershkowitz, C. R. Johnson: Spectra of matrices with P -matrix powers. Linear Algebra and its Applications 80, 159-171, **1986**.

PM -matrices are QM -matrices

If the eigenvalues of a QM -matrix are positive then this conjecture is true

Descartes Law of Signs:

If $\lambda \in \mathbb{R}$ is an eigenvalue of a Q -matrix then $\lambda > 0$

If $\lambda \in \mathbb{R}$ is an eigenvalue of a QM -matrix then $\lambda > 0$

Corollary:

If $\lambda \in \mathbb{R}$ is an eigenvalue of a P -matrix then $\lambda > 0$

If $\lambda \in \mathbb{R}$ is an eigenvalue of a PM -matrix then $\lambda > 0$

(together with R. Sendra and J. Sendra)

Characteristic polynomial of A^k :
the resultant of $P_A(Y)$ and $Y^k - T$ with respect to Y

EIGENVALUES OF A QM-MATRIX

A is a PM -matrix if A^k is a P -matrix for all positive integers k .

Are the eigenvalues of a PM -matrix necessarily positive?

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The conjecture is true for $n \leq 4$ since QM -matrices have this property

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The conjecture is true for $n \leq 4$ since QM -matrices have this property

If $\lambda \in \mathbb{C}$ is an eigenvalue of a QM -matrix A
then
 $|\lambda|$ is also an eigenvalue of A

EIGENVALUES OF A QM-MATRICES FOR N=3

Let A be a QM -matrix of size 3 over $\mathbb{R}$

The three eigenvalues of A are real numbers: all of them positive.

One eigenvalue real and two complex and non-real: λ , δ and $\bar{\delta}$.

$$\delta = \lambda (\cos(\alpha) + i \sin(\alpha))$$

$$P_{A^n}(T) = T^3 - \lambda^n (1 + 2 \cos(n\alpha)) T^2 + \lambda^{2n} (1 + 2 \cos(n\alpha)) T - \lambda^{3n}$$

EIGENVALUES OF A QM-MATRICES FOR N=3

Let A be a QM -matrix of size 3 over $\mathbb{R}$

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$$\delta = \lambda (\cos(\alpha) + i \sin(\alpha))$$

$$P_{A^n}(T) = T^3 - \lambda^n (1 + 2 \cos(n\alpha)) T^2 + \lambda^{2n} (1 + 2 \cos(n\alpha)) T - \lambda^{3n}$$

If A is a QM -matrix then $2 \cos(n\alpha) > -1$ for all $n \geq 1$. This implies

$$\alpha \in \bigcap_{n=1}^{\infty} \left(-\frac{2\pi}{3n}, \frac{2\pi}{3n} \right)$$

which is not possible since δ is complex and non-real.

Conjeture true for $n = 3$: eigenvalues of a PM -matrix are positive.

EIGENVALUES OF A QM-MATRICES FOR N=4

Let A be a QM -matrix of size 4 over $\mathbb{R}$

1. The four eigenvalues $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are real numbers: $\lambda_i > 0$.
2. Two of them are real, λ_1 and λ_2 , and two complex non-real, λ_3 and $\bar{\lambda}_3$: $\lambda_1 > 0, \lambda_2 > 0$.
3. The four eigenvalues are complex and non-real numbers: $\lambda_1, \bar{\lambda}_1, \lambda_2$ and $\bar{\lambda}_2$.

$$\lambda_3 = \rho (\cos(\alpha) + i \sin(\alpha)) \quad \rho = \lambda_1 \text{ or } \rho = \lambda_2$$

$$P_{A^n}(T) =$$

$$T^4 - (\lambda_1^n + \lambda_2^n + 2\rho^n \cos(n\alpha)) T^3 + (\rho^{2n} + \lambda_1^n \lambda_2^n + 2\rho^n \lambda_1^n \cos(n\alpha) + 2\rho^n \lambda_2^n \cos(n\alpha)) T^2 - \dots$$
$$\dots - \rho^n (2 \cos(n\alpha) \lambda_1^n \lambda_2^n + \rho^n \lambda_1^n + \rho^n \lambda_2^n) T + \rho^{2n} \lambda_1^n \lambda_2^n$$

EIGENVALUES OF A QM-MATRICES FOR N=4

$$\lambda_3 = \rho (\cos(\alpha) + i \sin(\alpha)) \quad \rho = \lambda_1 \text{ or } \rho = \lambda_2$$

If A is a QM -matrix then for all $n \geq 1$ we have

$$2 \cos(n\alpha) > -(L_1^n + L_2^n) \quad 2 \cos(n\alpha) > -\frac{1 + L_1^n L_2^n}{L_1^n + L_2^n} \quad 2 \cos(n\alpha) > -\frac{L_1^n + L_2^n}{L_1^n L_2^n}$$

where $L_1 = \frac{\lambda_1}{\rho}$ and $L_2 = \frac{\lambda_2}{\rho}$ (ie $L_1 = 1$ or $L_2 = 1$).

EIGENVALUES OF A QM-MATRICES FOR N=4

$$\lambda_3 = \rho (\cos(\alpha) + i \sin(\alpha)) \quad \rho = \lambda_1 \text{ or } \rho = \lambda_2$$

If A is a QM -matrix then for all $n \geq 1$ we have

$$2 \cos(n\alpha) > -(L_1^n + L_2^n) \quad 2 \cos(n\alpha) > -\frac{1 + L_1^n L_2^n}{L_1^n + L_2^n} \quad 2 \cos(n\alpha) > -\frac{L_1^n + L_2^n}{L_1^n L_2^n}$$

where $L_1 = \frac{\lambda_1}{\rho}$ and $L_2 = \frac{\lambda_2}{\rho}$ (ie $L_1 = 1$ or $L_2 = 1$).

Since $L_1 = 1$ or $L_2 = 1$ then $\alpha = 0$ which is not possible since λ_3 is complex and non-real:

$$\cos(n\alpha) > -\frac{1}{2} \text{ for all } n \implies \alpha \in \bigcap_{n=1}^{\infty} \left(-\frac{2\pi}{3n}, \frac{2\pi}{3n} \right)$$

Case discarded !

EIGENVALUES OF A QM-MATRICES FOR N=4

Let A be a QM -matrix of size 4 over $\mathbb{R}$

1. The four eigenvalues $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are real numbers: $\lambda_i > 0$.
2. Two of them are real, λ_1 and λ_2 , and two complex non-real, λ_3 and $\bar{\lambda}_3$: $\lambda_1 > 0, \lambda_2 > 0$.
3. The four eigenvalues are complex and non-real numbers: $\lambda_1, \bar{\lambda}_1, \lambda_2$ and $\bar{\lambda}_2$.

$$\lambda_1 = \rho (\cos(\alpha) + i \sin(\alpha)) \quad \lambda_2 = \sigma (\cos(\beta) + i \sin(\beta))$$

ρ and σ should be eigenvalues: this case does not happen

Case discarded !

Conjeture true for $n = 4$: eigenvalues of a PM -matrix are positive.

EIGENVALUES OF PM-MATRICES FOR $N=4$

A is a PM -matrix if A^k is a P -matrix for all positive integers k

Are the eigenvalues of a PM -matrix necessarily positive?

D. Hershkowitz, C. R. Johnson: Spectra of matrices with P -matrix powers. Linear Algebra and its Applications 80, 159-171, **1986**.

The conjecture is true for $n \leq 4$

since

QM -matrices have this property

EIGENVALUES OF A QM-MATRICES FOR N=5

Let A be a QM -matrix of size 5 over $\mathbb{R}$

Three possibilities:

1. If all eigenvalues are real numbers then they are positive.
2. $\rho > 0, \rho (\cos(\alpha) \pm i \sin(\alpha)), \lambda > 0, \tau > 0$ and $\alpha \neq 0$.
3. $\rho > 0, \rho (\cos(\alpha) \pm i \sin(\alpha)), \rho (\cos(\beta) \pm i \sin(\beta)), \alpha \neq 0, \beta \neq 0$.

$$P_{A^n}(T) =$$

$$\begin{aligned} & T^5 - T^4 (2\rho^n \cos(n\alpha) + \lambda^n + \rho^n + \tau^n) + \\ & + T^3 (\lambda^n \rho^n + \lambda^n \tau^n + \rho^{2n} + \rho^n \tau^n + 2\lambda^n \rho^n \cos(n\alpha) + 2\rho^{2n} \cos(n\alpha) + 2\rho^n \tau^n \cos(n\alpha)) + \\ & - T^2 \rho^n (2\lambda^n \rho^n \cos(n\alpha) + 2\lambda^n \tau^n \cos(n\alpha) + 2\rho^n \tau^n \cos(n\alpha) + \lambda^n \rho^n + \lambda^n \tau^n + \rho^{2n} + \rho^n \tau^n) + \\ & + T \rho^{2n} (2\lambda^n \tau^n \cos(n\alpha) + \lambda^n \rho^n + \lambda^n \tau^n + \rho^n \tau^n) - \lambda^n \rho^{3n} \tau^n \end{aligned}$$

$$L_1 = \frac{\lambda}{\rho} \text{ and } L_2 = \frac{\tau}{\rho}$$

$$\text{for all } n \geq 1 \quad 2 \cos(n\alpha) > -1 - \min \left\{ L_1^n + L_2^n, \frac{L_1^n L_2^n}{L_1^n + L_2^n + 1}, \frac{1}{L_1^n + L_2^n + L_1^n L_2^n}, \frac{1}{L_1^n} + \frac{1}{L_2^n} \right\}$$

If $L_1 > 1$ and $L_2 > 1$ then

$$\min \left\{ L_1^n + L_2^n, \frac{L_1^n L_2^n}{L_1^n + L_2^n + 1}, \frac{1}{L_1^n + L_2^n + L_1^n L_2^n}, \frac{1}{L_1^n} + \frac{1}{L_2^n} \right\} = \frac{1}{L_1^n + L_2^n + L_1^n L_2^n}$$

And if $L = \min\{L_1, L_2\}$ then

$$\left\{ \alpha : \forall n \quad \cos(n\alpha) > -\frac{1}{2} - \frac{1}{2} \frac{1}{L_1^n + L_2^n + L_1^n L_2^n} \right\} \subseteq \subseteq \left\{ \alpha : \forall n \quad \cos(n\alpha) > -\frac{1}{2} - \frac{1}{2} \frac{1}{L^n} \right\}$$

$$\begin{aligned}
& \bigcap_{k=1}^n \left\{ \alpha: \cos(k\alpha) > -\frac{1}{2} - \frac{1}{2} \frac{1}{L^k} \right\} = \\
& = \left(-\frac{2 \left\lfloor \frac{n-1}{3} \right\rfloor + 1}{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 1} \pi + \frac{\arccos \left(\frac{1 + L^{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 1}}{2L^{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 1}} \right)}{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 1}, -\frac{2 \left\lfloor \frac{n-2}{3} \right\rfloor + 1}{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 1} \pi + \frac{\arccos \left(\frac{1 + L^{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2}}{2L^{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2}} \right)}{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2} \right) \\
& \quad \cup \left(-\frac{\pi}{n} + \frac{1}{n} \arccos \left(\frac{L^n + 1}{2L^n} \right), \frac{\pi}{n} - \frac{1}{n} \arccos \left(\frac{L^n + 1}{2L^n} \right) \right) \cup \\
& \quad \cup \left(\frac{2 \left\lfloor \frac{n-2}{3} \right\rfloor + 1}{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 1} \pi + \frac{\arccos \left(\frac{1 + L^{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2}}{2L^{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2}} \right)}{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2}, \frac{2 \left\lfloor \frac{n-1}{3} \right\rfloor + 1}{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 1} \pi + \frac{\arccos \left(\frac{1 + L^{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 2}}{2L^{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 2}} \right)}{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 2} \right) =
\end{aligned}$$

$$\begin{aligned}
& \bigcap_{k=1}^n \left\{ \alpha: \cos(k\alpha) > -\frac{1}{2} - \frac{1}{2} \frac{1}{L^k} \right\} = \\
& = \left(-\frac{2 \left\lfloor \frac{n-1}{3} \right\rfloor + 1}{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 1} \pi + \frac{\arccos \left(\frac{1 + L^{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 1}}{2L^{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 1}} \right)}{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 1}, -\frac{2 \left\lfloor \frac{n-2}{3} \right\rfloor + 1}{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 1} \pi + \frac{\arccos \left(\frac{1 + L^{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2}}{2L^{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2}} \right)}{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2} \right) \\
& \quad \cup \left(-\frac{\pi}{n} + \frac{1}{n} \arccos \left(\frac{L^n + 1}{2L^n} \right), \frac{\pi}{n} - \frac{1}{n} \arccos \left(\frac{L^n + 1}{2L^n} \right) \right) \cup \\
& \quad \cup \left(\frac{2 \left\lfloor \frac{n-2}{3} \right\rfloor + 1}{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 1} \pi + \frac{\arccos \left(\frac{1 + L^{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2}}{2L^{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2}} \right)}{3 \left\lfloor \frac{n-2}{3} \right\rfloor + 2}, \frac{2 \left\lfloor \frac{n-1}{3} \right\rfloor + 1}{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 1} \pi + \frac{\arccos \left(\frac{1 + L^{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 2}}{2L^{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 2}} \right)}{3 \left\lfloor \frac{n-1}{3} \right\rfloor + 2} \right) = \\
& \quad = \left\{ -\frac{2\pi}{3}, 0, \frac{2\pi}{3} \right\}
\end{aligned}$$

EIGENVALUES OF A QM-MATRICES FOR N=5

The remaining cases are solved in a similar way and produce exactly the same solution: A is a QM -matrix with only one complex and non real eigenvalue iff $\alpha = \pm \frac{2\pi}{3}$.

Are the eigenvalues of a PM -matrix necessarily positive?

D. Hershkowitz, C. R. Johnson: Spectra of matrices with P-matrix powers. Linear Algebra and its Applications 80, 159-171, **1986**.

For $n = 5$

QM -matrices do not have this property

EIGENVALUES OF A QM-MATRICES FOR N=5

Let A be a QM -matrix of size 5 over $\mathbb{R}$

Three possibilities:

1. If all eigenvalues are real numbers then they are positive.
2. $\rho > 0, \rho (\cos(\alpha) \pm i \sin(\alpha)), \lambda > 0, \tau > 0, \alpha \neq 0$.
3. $\rho > 0, \rho (\cos(\alpha) \pm i \sin(\alpha)), \rho (\cos(\beta) \pm i \sin(\beta)), \alpha \neq 0, \beta \neq 0$.

$$\begin{aligned} P_{A^n}(T) = & T^5 - T^4 \rho^n \left(1 + 2 \cos(n\alpha) + 2 \cos(n\beta) \right) + \\ & + 2T^3 \rho^{2n} \left(1 + 2 \cos(n\alpha) \cos(n\beta) + \cos(n\beta) + \cos(n\alpha) \right) + \\ & - 2T^2 \rho^{3n} \left(1 + 2 \cos(n\alpha) \cos(n\beta) + \cos(n\beta) + \cos(n\alpha) \right) + \\ & + T \rho^{4n} \left(1 + 2 \cos(n\alpha) + 2 \cos(n\beta) \right) - \rho^{5n} \end{aligned}$$

EIGENVALUES OF A QM-MATRICES FOR N=5

A is a QM -matrix iff for all $n \geq 1$

$$1 + 2 \cos(n\alpha) + 2 \cos(n\beta) > 0$$

$$1 + 2 \cos(n\alpha) \cos(n\beta) + \cos(n\beta) + \cos(n\alpha) > 0$$

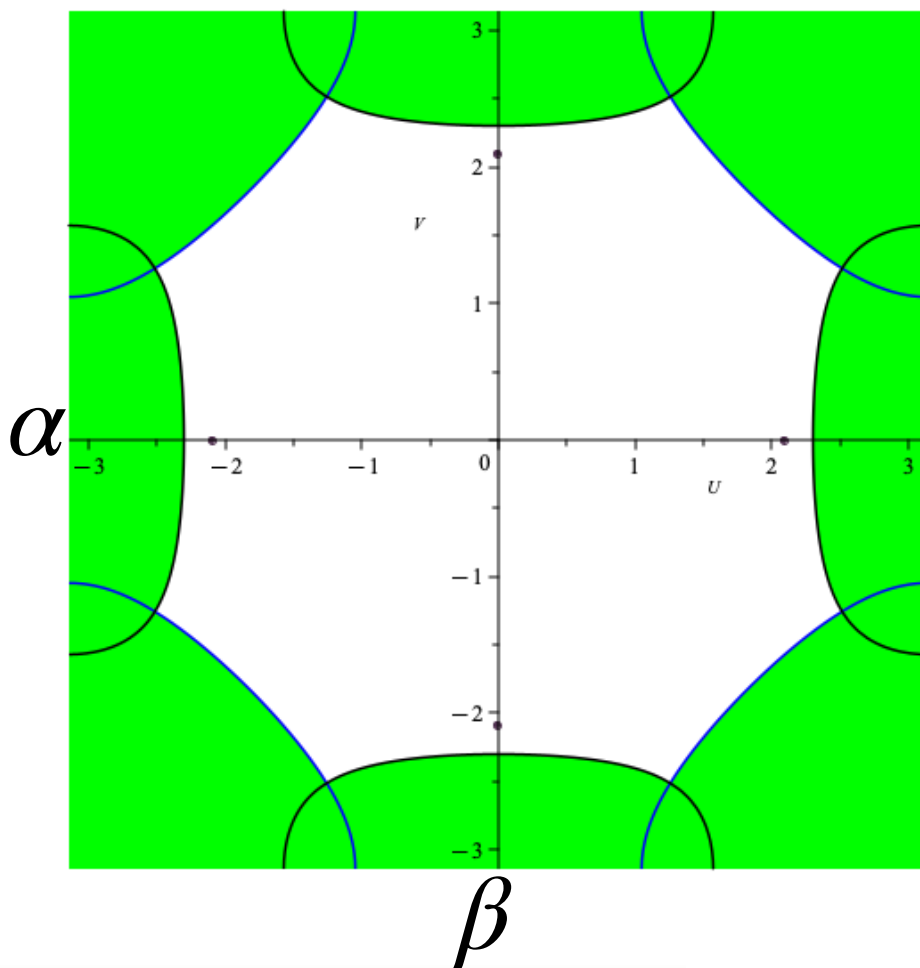
EIGENVALUES OF A QM-MATRICES FOR N=5

A is a QM -matrix iff for all $n \geq 1$

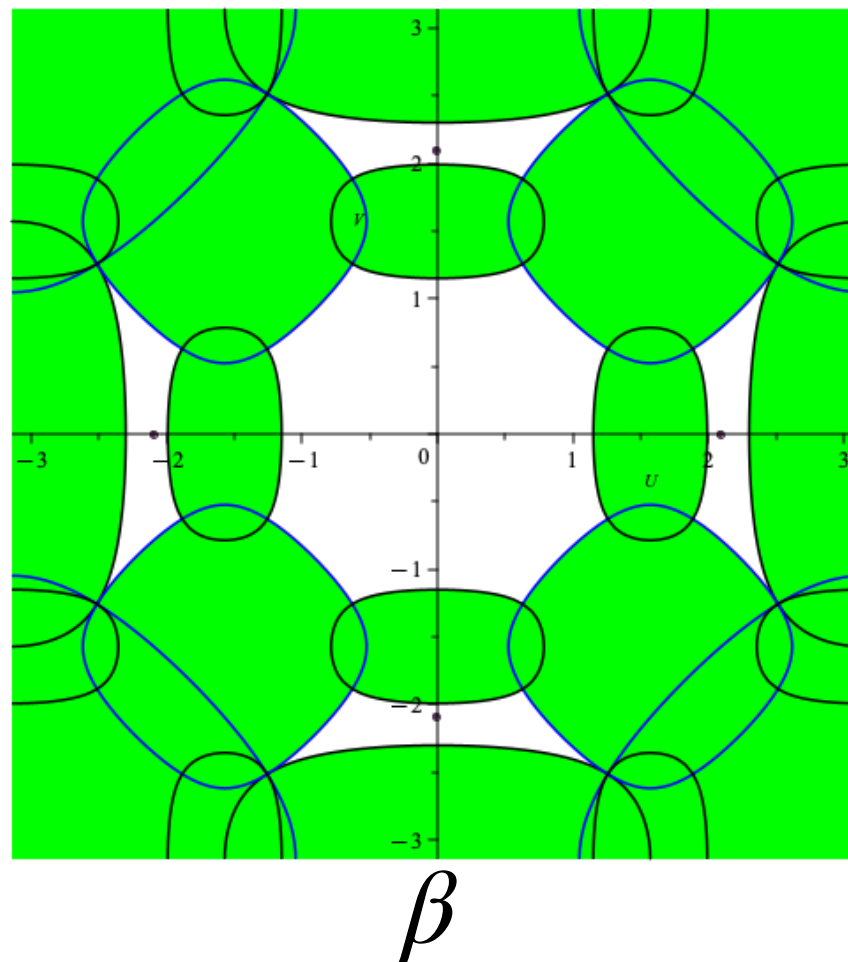
$$1 + 2 \cos(n\alpha) + 2 \cos(n\beta) > 0$$

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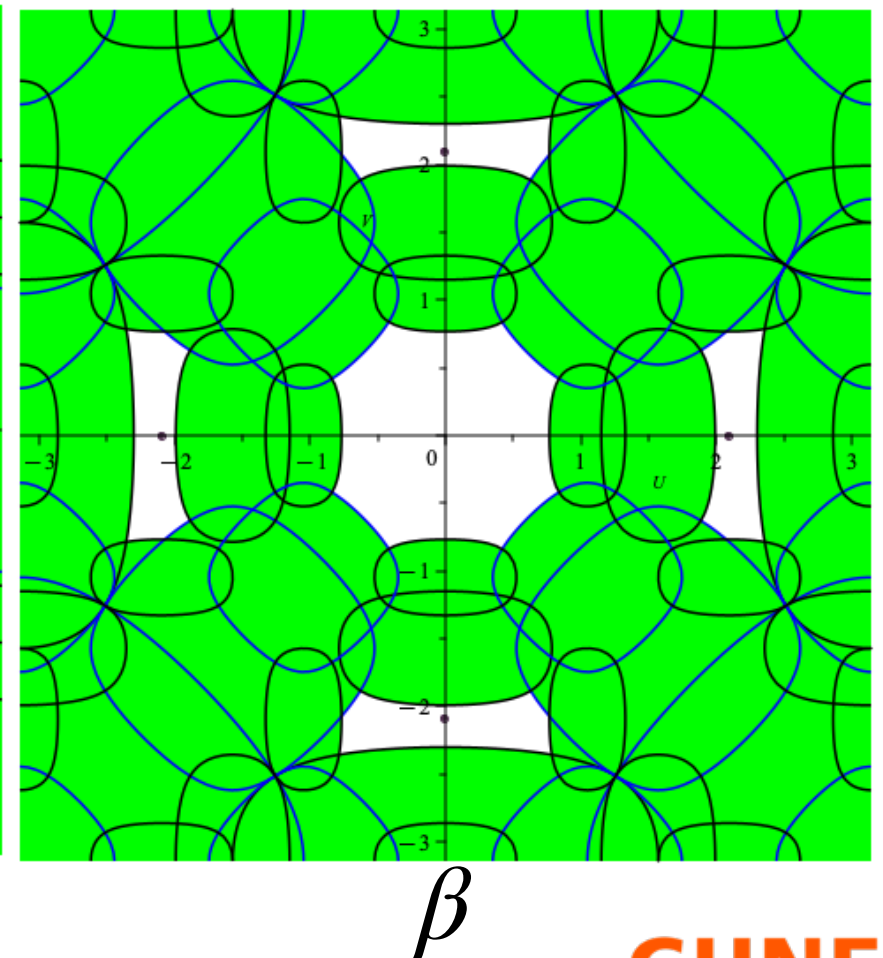
$n = 1$



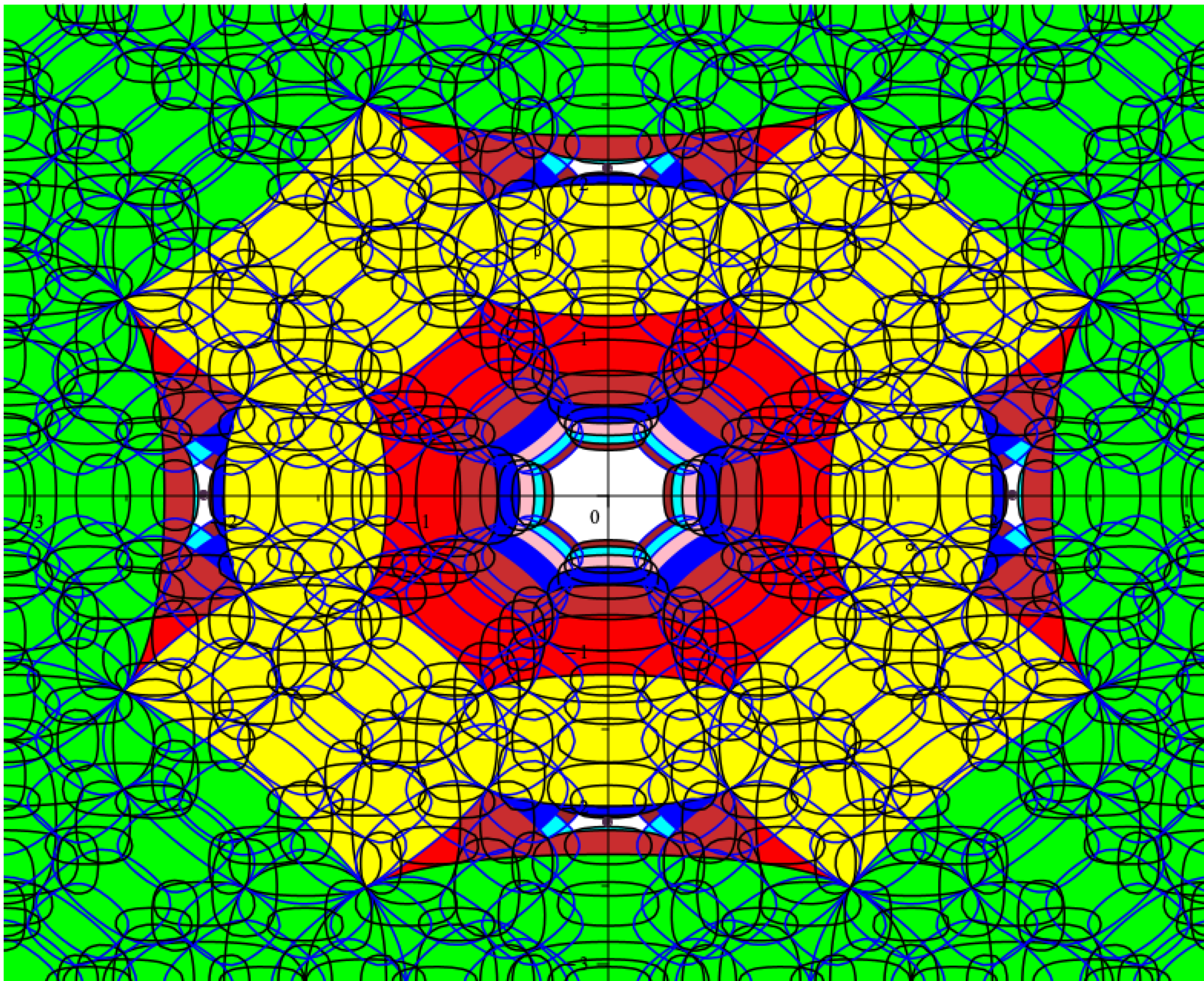
$n \leq 2$



$n \leq 3$



$$n \leq 8$$

 α

 β

$$n \leq 8$$

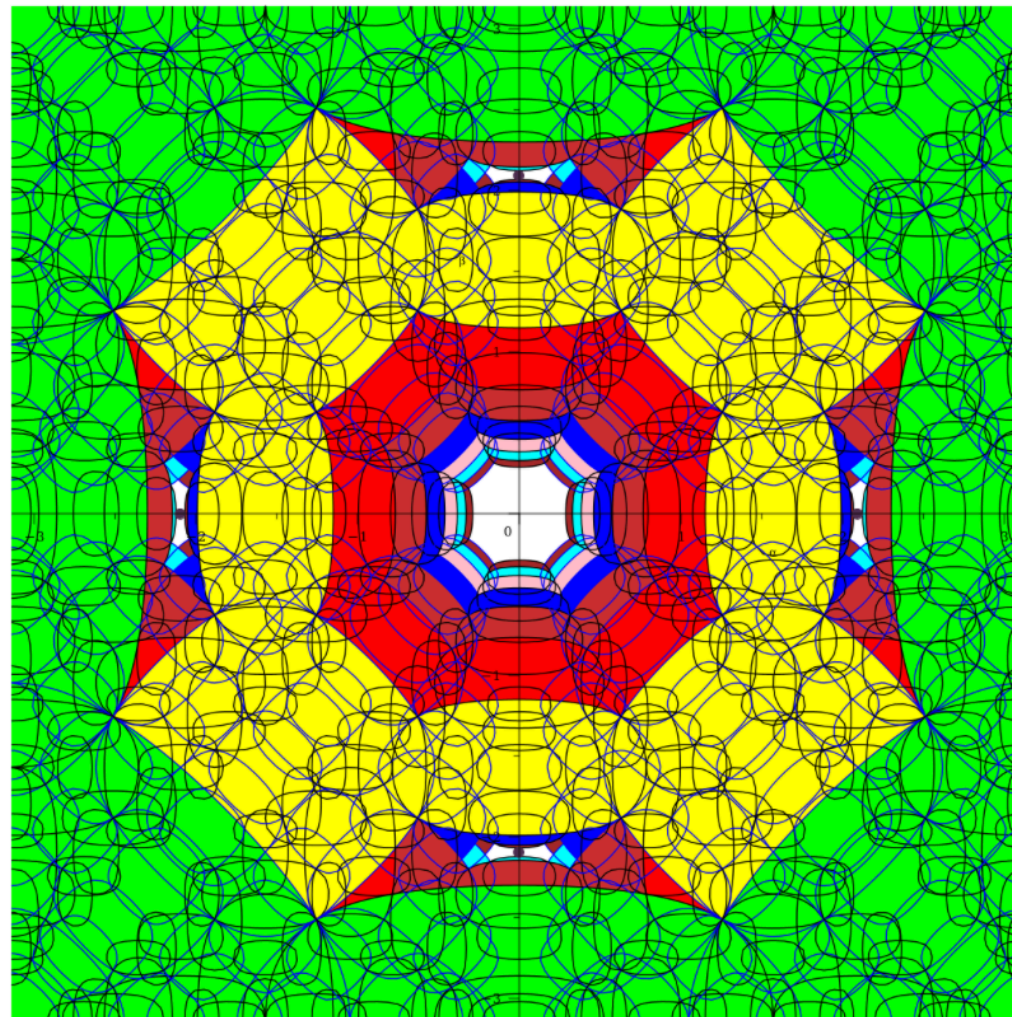
 α

In the limit, either $\alpha = 0$ or $\beta = 0$

 β

October 28 2024

6 ★



Arguments (α and β) of the two complex eigenvalues when A^k is a QM-matrix for $1 \leq k \leq 8$.

Let A be a real square matrix of size n . A is a QM-matrix if A^k is a QM-matrix for all $k \geq 1$. A is a QM-matrix if the sum of the k -by- k principal minors is positive ($1 \leq k \leq n$).

Let A be a real square matrix of size 5 with one real eigenvalue and two complex and non real eigenvalues with arguments α and β . A is a QM-matrix iff for all $n \geq 1$ we have $1 + 2 \cos(n\alpha) + 2 \cos(n\beta) > 0$ and $1 + 2 \cos(n\alpha) \cos(n\beta) + \cos(n\beta) + \cos(n\alpha) > 0$.

The figure represents (white area) the values of α and β satisfying the two previous conditions for $n=1,2,\dots,8$. Final conclusion says that such a matrix does not exist if α and β are different and not zero.

Code included:

```
display([
implicitplot([
```

EIGENVALUES OF A QM-MATRICES FOR N=5

Three possibilities:

1. If all eigenvalues are real numbers then they are positive.
2. if $\rho > 0, \rho (\cos(\alpha) \pm i \sin(\alpha)), \lambda > 0, \tau > 0 : \alpha = \pm \frac{2\pi}{3}$.
3. ~~$\rho > 0, \rho (\cos(\alpha) \pm i \sin(\alpha)), \rho (\cos(\beta) \pm i \sin(\beta))$~~ .

Are the eigenvalues of a *PM*-matrix necessarily positive?

D. Hershkowitz, C. R. Johnson: Spectra of matrices with P-matrix powers. Linear Algebra and its Applications 80, 159-171, **1986**.

For $n = 5$, *QM*-matrices do not have this property

MAIN QUESTION TO ANSWER

A is a PM -matrix if A^k is a P -matrix for all positive integers k

Are the eigenvalues of a PM -matrix necessarily positive?

D. Hershkowitz, C. R. Johnson: Spectra of matrices with P -matrix powers. Linear Algebra and its Applications 80, 159-171, **1986**.

If A is a P -matrix then A is a Q -matrix.

If A is a PM -matrix then A is a QM -matrix.

But, to study the properties of the eigenvalues of a QM -matrix is still an interesting question: $n \geq 6$?

MAIN QUESTION TO ANSWER

A is a PM -matrix if A^k is a P -matrix for all positive integers k

Are the eigenvalues of a PM -matrix necessarily positive?

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If A is a P -matrix then A is a Q -matrix.

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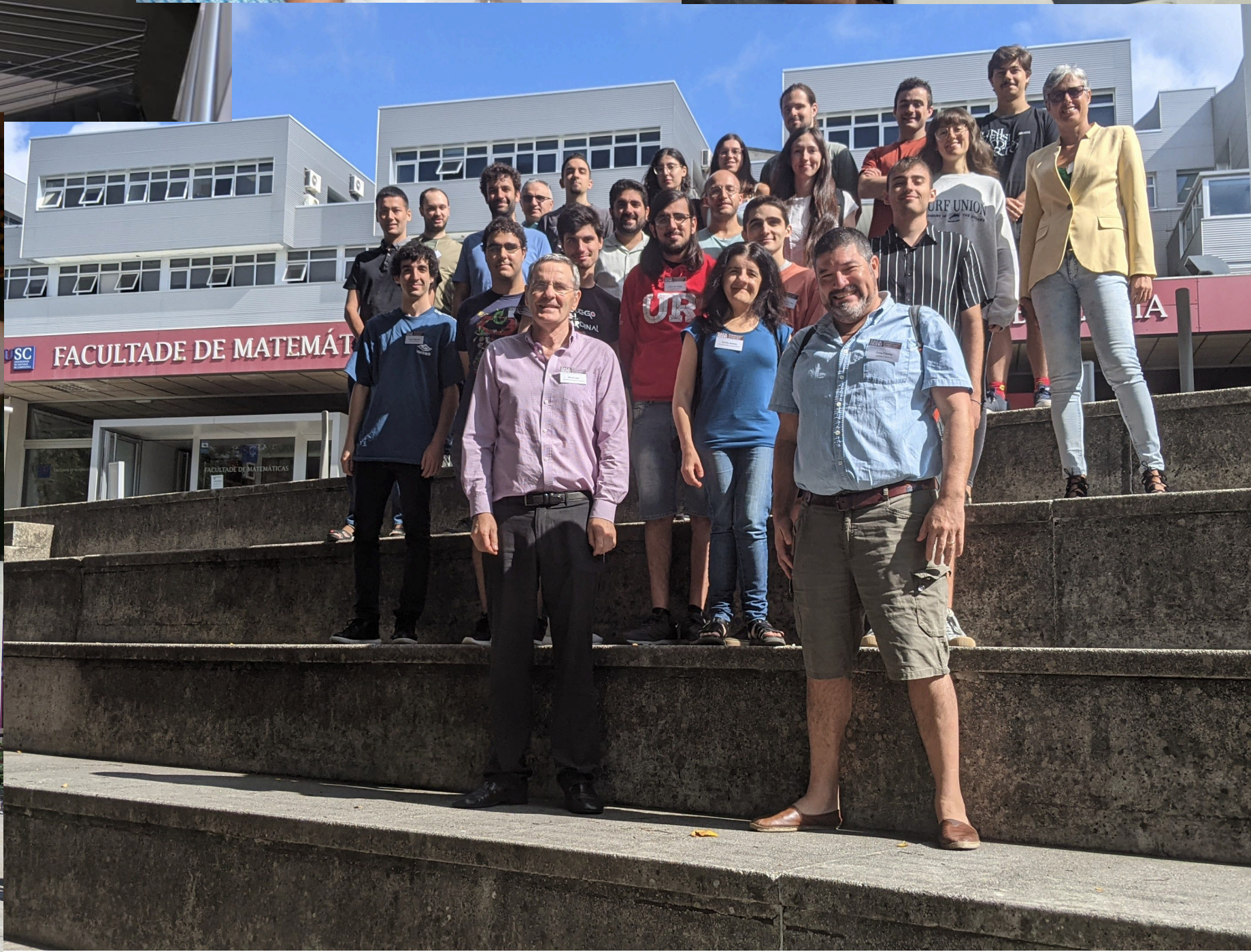
The eigenvalues of a PM -matrix are necessarily positive

S. Mondal, K. C. Sivakumar, M. J. Tsatsomeros: P -matrix powers. Linear Algebra and its Applications 699, 355-366, **2024**.

But, to study the properties of the eigenvalues of a QM -matrix is still an interesting question: $n \geq 6$?

**NO
CONCLUSIONS
BUT
A SMALL PRESENT FROM
THE EACA NETWORK**

WE WILL MISS YOU







Plesase do not forget us !

Thanks Manolo !

¡ Gracias Manolo !

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